Warsaw Summer School 2023, OSU Study Abroad Program

Difference Between Means

Hypotheses

A **<u>hypothesis</u>** = a prediction about the relationship between variables. It is usually based upon theoretical expectations about how things work.

An empirical hypothesis is a **<u>testable guess</u>** or tentative proposition given to explain some data. Because we use **<u>probability</u>** to make decision about it, this guess can be proved or disproved in a **<u>statistical sense</u>** and not in formal sense - as in mathematics or logic.

Hypotheses

Hypotheses come always in a pair:

The <u>Null hypothesis</u> (Ho) = a statement about parameters that is usually the opposite of the researcher' belief.

(Ho) states that there is <u>no change</u>, <u>no difference</u>, or <u>no relationship</u>. Thus, Ho always has the equal sign.

The <u>Alternative</u> (substantive/research) <u>hypothesis</u> (H1): expresses the researcher's belief/ expectation.

Six steps of testing hypotheses

1) Assume random sample

2) State Ho and H1

3) Specify the sampling distribution & the test statistic

4) Choose alpha level & specify the rejection region in picture of the sampling distribution.

5) Compute test statistic from data.

6) Make decision & interpret results.

Examples

Problem A: Is there a difference in the mean number of elections Ohio voters have participated in compared to voters in the US?

Hypotheses:

- H0: μ (Ohio) = μ (US) ,   H1: μ (Ohio) $\neq$ μ (US)

Problem B: Is there a difference in the mean number of classes that male students take in a semester compared to female students?

- H0: μ (males) = μ (females), H1: μ (males) $\neq$ μ (females)

Hypotheses

- **H0: μ (males) = μ (females)**
  - **is the same as**
- **H0: μ (males) - μ (females) = 0**

- **H1: μ (males) $\neq$ μ (females)**
  - **is not the same as**
- **H1: μ (males) > μ (females) or μ (males) < μ (females)**

Rejection of the null hypothesis

When can we reject Ho that there is no difference between two means?

We can reject the null hypothesis if the standardized difference between means is <u>significantly</u> large

Two issues:

- standardization of the difference

- assessment of the magnitude of the difference

Standard error of the difference between means

Test of the difference between two means
equals to
the difference between both means
relative to
the standard error of the difference between these two means

$$z/t = \frac{Mean(I) - Mean(II)}{Standard\ error\ of\ [Mean(I) - Mean(II)]}$$

Mean(I) - Mean(II)
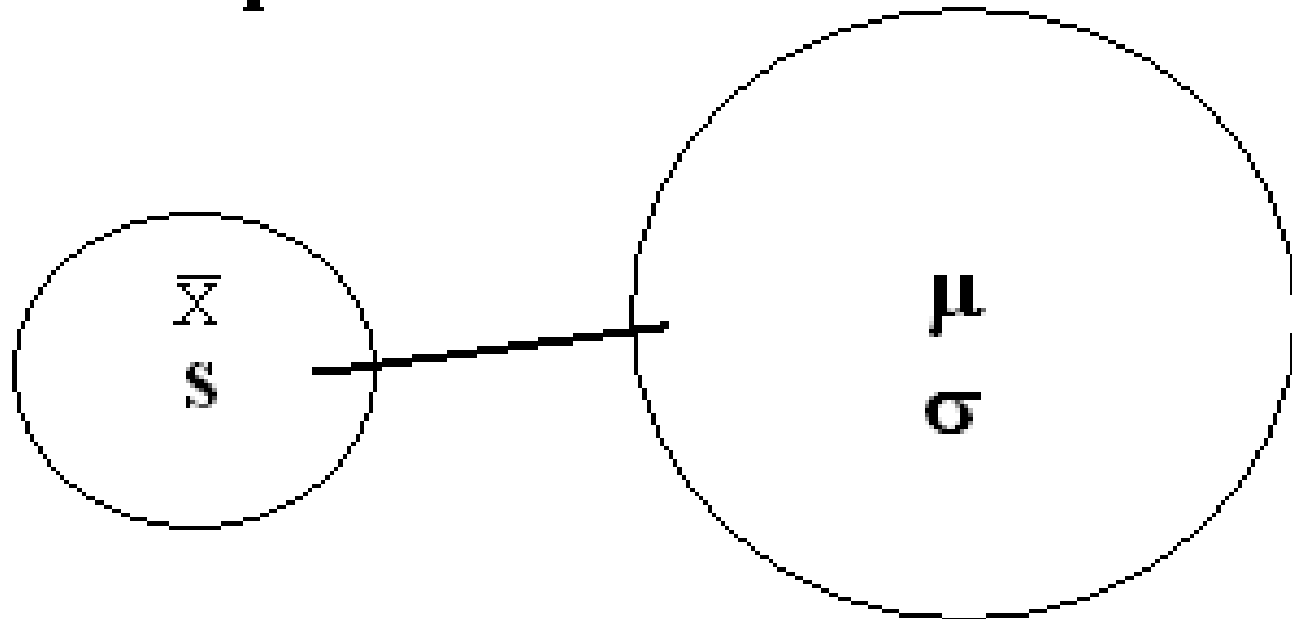
Situation A

Mean (I) = mean value from the sample $\overline{X}$

Mean (II) = mean value in the population μ

Sample Mean vs. Population Mean

Situation A

Sample

Population

$$\bar{x}$$
$$s$$

$$\mu$$
$$\sigma$$

$$Ho: \quad \bar{x} - \mu = 0$$

Standard error of Mean(I) - Mean(II)

Two possibilities:

A1 Standard error for $\overline{X} - \mu$ with known σ

$$\sigma_x = \sigma / \sqrt{n}$$

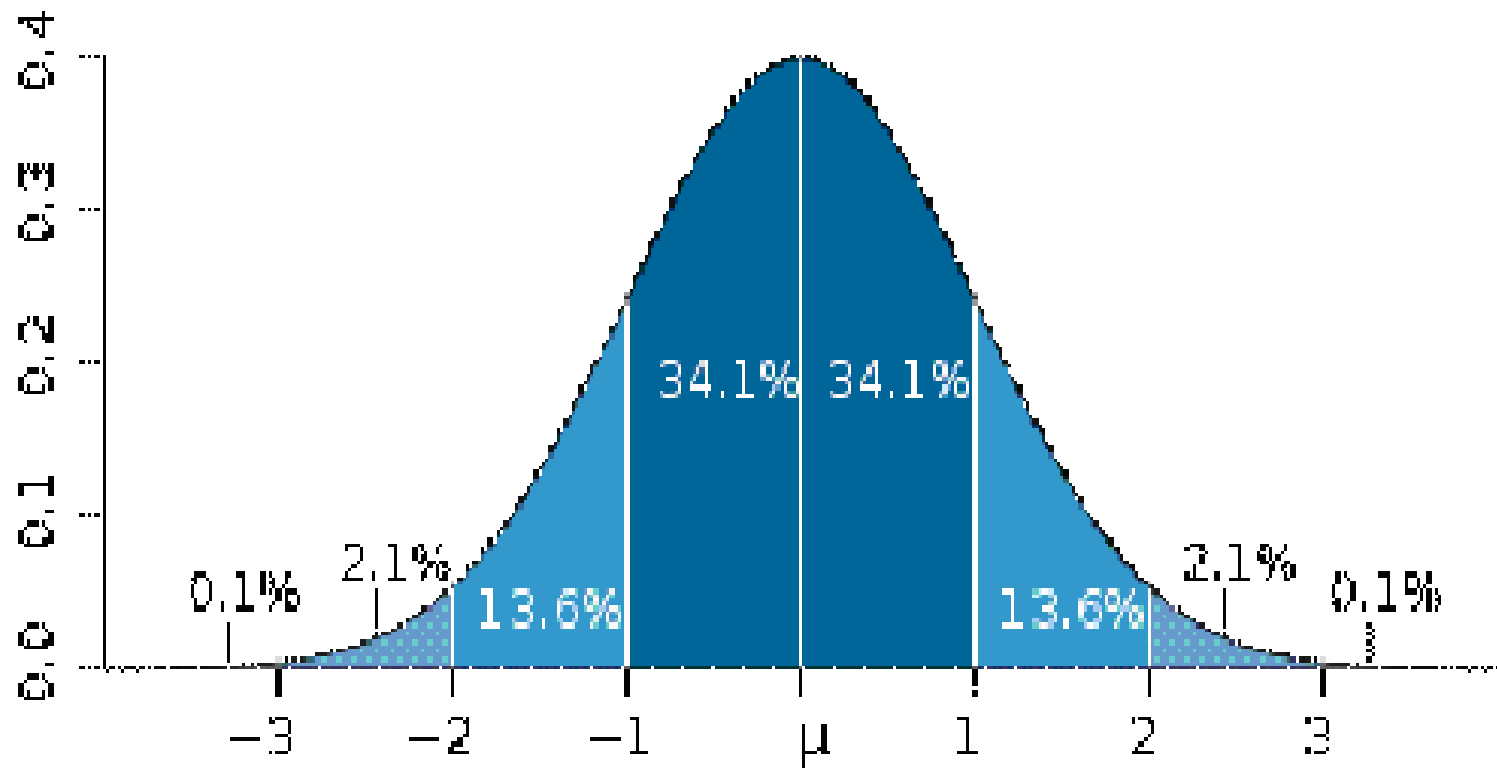**A2 Standard error for $\overline{X} - \mu$ without known σ**

$$s_x = s / \sqrt{n-1}$$

Standard error of Mean(I) - Mean(II)

A1 Standard error for $\overline{X} - \mu$ with known σ

$$\sigma_x = \sigma / \sqrt{n}$$

$$Z = \frac{\overline{X} - \mu}{\sigma_x}$$

z-test distribution

Example

$\mu = 40 \quad \sigma = 5 \qquad \bar{X} = 42 \qquad N = 100$

1.   Assumptions

2. State your hypotheses: H0: $\bar{X} = \mu$, H1: $\bar{X} \neq \mu$ (H1: $\bar{X} \neq 40$)

3. Sampling distribution & the test statistic (here z-test)

4. Alpha level $p < 0.05$ (95%)

5. Test statistic

(Is the difference 42 - 40 simply due to sampling error).

$$\text{z-test} = (42 - 40) / (5/\sqrt{100}) = 2 / .5 = 4$$

6. Decision & interpret results

σ unknown

This example refers to a one-sample z-test: We compare one sample to the population information given, to see if the sample mean (x) is different enough from the population mean (μ) to say that the sample is distinct from the population.

Often the population standard deviation (σ) is not provided. Then, we cannot use the z-test because we do not know that the sampling distribution is actually normal. All we have is sample information (x, s) a given population mean (μ).

Standard error of Mean(I) - Mean(II)

A1 Standard error for $\overline{X} - \mu$ with known σ

$$\sigma_x = \sigma / \sqrt{n} \qquad\qquad Z = \frac{\overline{X} - \mu}{\sigma_{\overline{x}}}$$

A2 Standard error for $\overline{X} - \mu$ with unknown σ

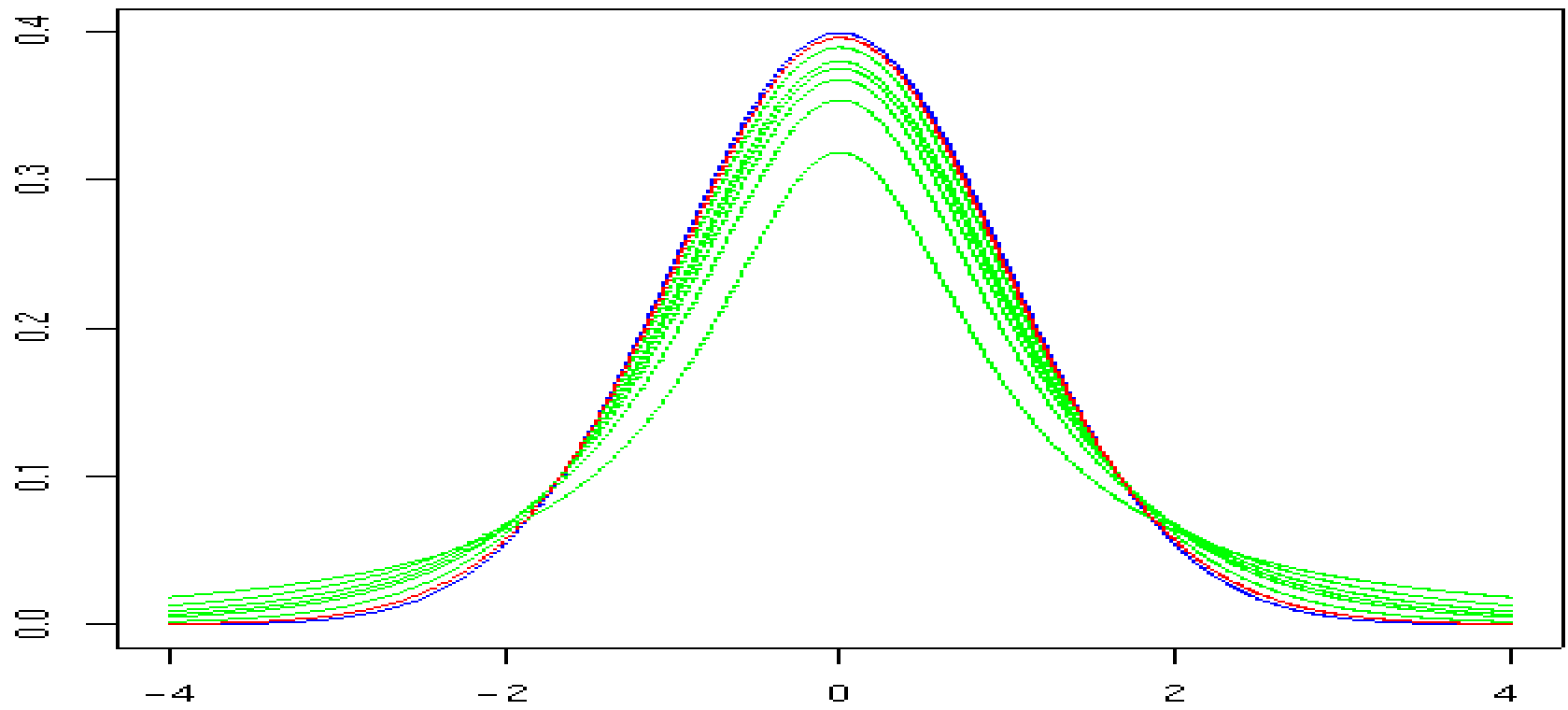$$s_x = s / \sqrt{n-1} \qquad\qquad t = \frac{\overline{X} - \mu}{s_{\overline{x}}}$$

t-distribution

1. **T-distribution varies with degrees of freedom of the sample**

2. **For small n, it is flatter than z-distribution, although also unimodal symmetric, mesokurtosic**

3. **For large n, t-distribution and z-distribution converge**

t-distribution
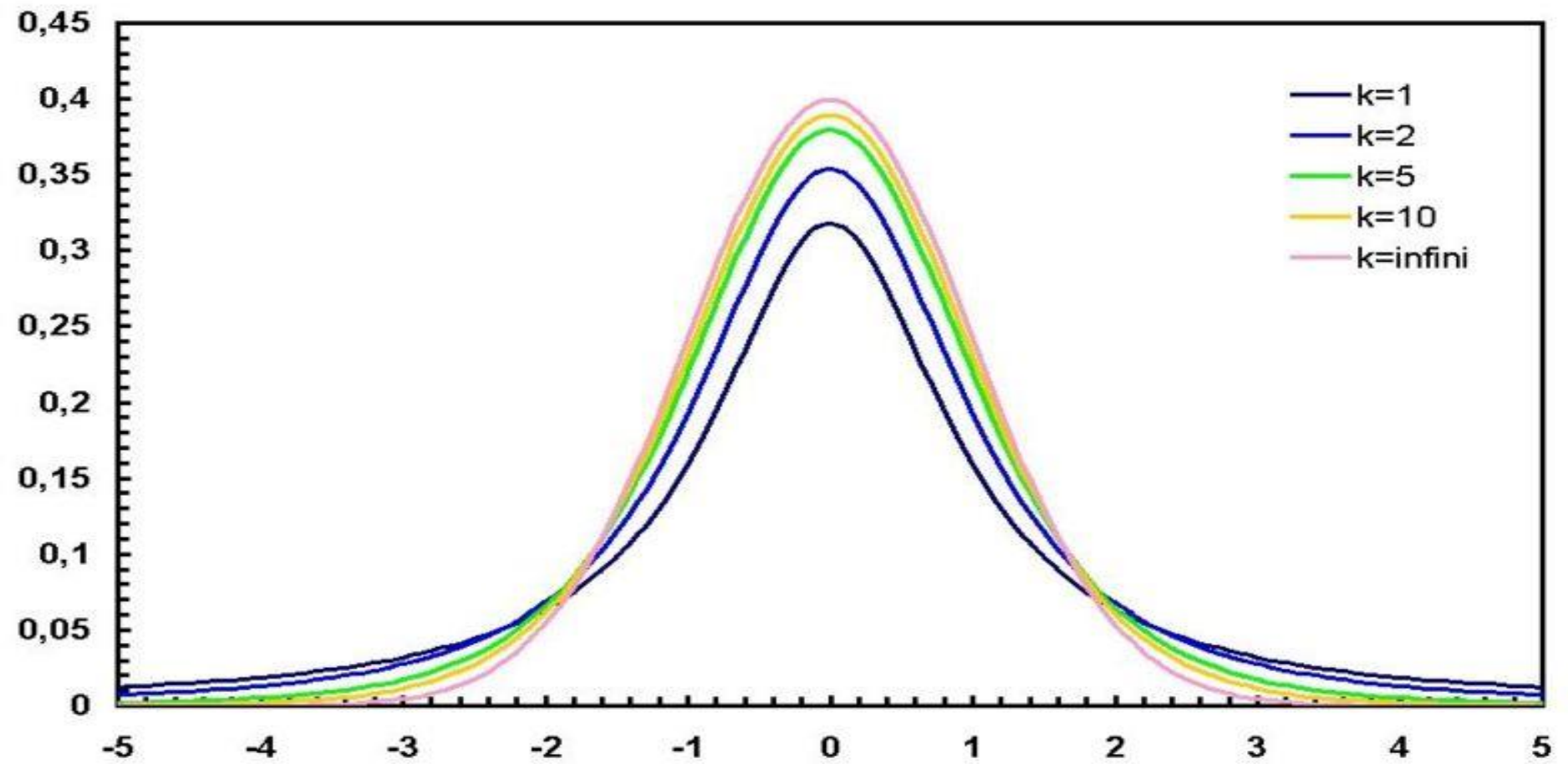
t-distribution

Example

E.g.: For a sample of OSU students compare the average number of months spent looking for jobs after graduation to national average for all new college graduates.

$$\mu = 4 \qquad \bar{X} = 3.6 \qquad s = 2.1 \qquad N = 100$$

1) **Assumptions?**

2) **Hypotheses?** **H0: X= $\mu = 4$ or H0: X - $\mu = 0$;**
   H1: X $\neq \mu \neq 4$

3) **The standard error of the population is not available. Thus, use T-test.**

4) **Choose an alpha level (1 minus the level of confidence (95%)**

   The critical values are the t-values that define the rejection region. Use Table C to find critical values for t.To do so, we need to know:

   -whether we are using a 2-tailed test or a 1-tailed test (2 tailed now),

   - the degrees of freedom $\rightarrow$ for one-sample t-tests,
   df = N – 1

5) **Calculate t-statistic**

$$t = \frac{\overline{X} - \mu}{s_{\overline{x}}} \qquad\qquad s_{\overline{x}} = \frac{s}{\sqrt{N-1}}$$

t-test = (3.6 – 4) / (2.1 / √ 99) = -. 4 / .211 = -1.896

6) Make decision (fail to reject the null)

Whenever t-calculated > t-critical (from the table) we reject Ho

Interpretation: There is no evidence to suggest that OSU graduates differ from other graduates with respect of the time looking for a job.
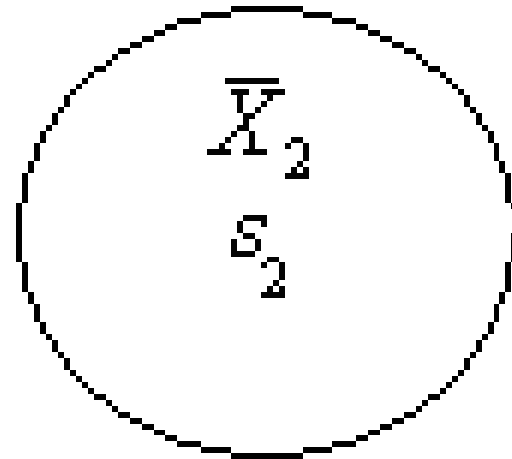
Two-sample test

If we compare two samples, to see whether there is a difference between the two groups, we use a two-sample t-test.

Sample 1   Sample 2

$$\overline{X}_1$$

$$s_1$$

$$\overline{X}_2$$

$$s_2$$
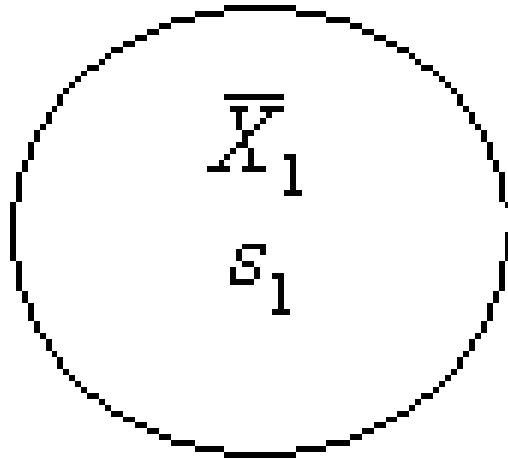
$$\text{Ho:} \quad \overline{x}_1 - \overline{x}_2 = 0$$

Testing

- .

$$t = \frac{\text{Mean(I)} - \text{Mean(II)}}{\text{Standard error of [Mean(I) - Mean(II)]}}$$

$$t = \frac{\overline{X}_1 - \overline{X}_2}{s_{\overline{X}1-\overline{X}2}}$$

$$s_{\overline{X}1-\overline{X}2} = \sqrt{\left(\frac{N_1 s_1^2 + N_2 s_2^2}{N_1 + N_2 - 2}\right)\left(\frac{N_1 + N_2}{N_1 N_2}\right)}$$

Example

Compare the average number of months spent looking for jobs after graduation in our sample of OSU students and a sample of students from Harvard College.

$\overline{X}_{OSU} = 3.6$ $\overline{X}_{HC} = 2.7$

$s_{OSU} = 2.1$ $s_{HC} = 2.3$

$N_{OSU} = 100$ $N_{HC} = 100$

Example

1) **Assumptions**
2) **Null and research hypotheses**

$$Ho: \bar{X}_{OSU} = \bar{X}_{HC} \text{ is equivalent to } Ho: \bar{X}_{OSU} - \bar{X}_{HC} = 0$$

$$H1: \bar{X}_{OSU} \neq \bar{X}_{HC}$$

3) **Specify sampling distribution. We use t-test.**
4) **Choose an alpha level & specify rejection region.**
   For two-sample t-test, df = N1+N2-2;
   here: df=100+100-2 = 198
   Look up t-critical values in table C , two-tailed: +/- 1.96
   for α = .05, or 95%

Example

5) Compute test statistic

$$s_{\overline{x}_1 - \overline{x}_2} = \sqrt{\left(\frac{100(2.1)^2 + 100(2.3)^2}{100 + 100 - 2} \right) \left(\frac{100 + 100}{100(100)} \right)} = .313$$

t-calculated = (3.6-2.7) / .313 = .9/.313 = 2.875

6) Make decision & interpret results
Reject the null.